THE ENGLISH OF THE PSYCHOPHYSICAL MEASUREMENT METHODS.

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For some years past I have had it in mind to issue for class use a series of four-page leaflets, giving the schemata of the principal measurement methods (just noticeable stimuli, minimal changes, equivalents, mean gradations, average error, right and wrong cases) with concrete illustrations. My aim would be to subordinate mathematics to psychological analysis ; to show the introspective reason for each step in each method, but to make the mathematical treatment of the results a matter of rote. The mathematical basis of the error methods might then be set forth in a separate appendix to the six leaflets, should there be any demand for it.

The carrying out of this idea has been delayed by the lack of English symbols for the exposition of the methods. In the translator's preface to Kuelpe's "Outlines of Psychology" (1895) I wrote: "Many English writers on psychophysical questions have employed the German abbreviations ; there are obvious difficulties in the way of an English terminology; and it is not probable that any proposals which could be made at the present time would find general acceptance." I hoped that, if attention were thus called to a serious defect in our working apparatus, some one might come forward with suggestions for a remedy. The hope has not been fulfilled. I therefore venture to offer some suggestions of my own. I do so because I feel that leaflets of the kind just described would be of real service in my own class work, and presumably in that of other psychologists.

Sensitivity and sensible discrimination can evidently be shortened, as they have been in some of the periodical summaries in *Mind*, to S and SD. Sensation must be represented by s. There is then every reason to keep the German r as the symbol for stimulus ; there can be but few students who have not come across an r used in this sense, and r comes before s in the order of the alphabet as stimulus before

sensation in the order of the experiment. A difference between two stimuli, any $r-r_1$, will be D; the just noticeable difference or difference limen $\triangle r$. The stimulus limen or just noticeable stimulus may then be termed r_λ. We thus have r_λ as the measure of sensitivity,[1] and $\triangle r$ and $\dfrac{\triangle r}{r}$ as the measures of the absolute and relative SD respectively. The standard stimulus I call U (uniform), the stimulus of comparison V (variable) ; so that $\dfrac{\triangle r}{r}$ becomes, in practice, $\dfrac{\triangle r}{U}$. The mean variation is mv, written as the case requires mv, $\dfrac{mv}{r}$, $\dfrac{mv}{U}$, or $\dfrac{mv}{D}$; the mean or average error is me.

Paralleling these symbols with those given by Kuelpe in his Table on pages 52, 53, we have:

German.	English.	
$r, r_1, r_2, r_3, \ldots$	$r, r_1, r_2, r_3, \ldots$	Stimulus.
$\triangle r$ or D or $r\text{-}r_1, \ldots$	D or $r\text{-}r_1, \ldots$	Stimulus difference.
S or j. n. $\triangle r$,	$\triangle r$,	Difference limen.
German S, or j. n. r, r_λ,		Stimulus limen.
$\triangle r$, S,	$\triangle r$,	Abs. mag. of SD.
$\dfrac{\triangle r}{r}$, $\dfrac{S}{r}$,	$\dfrac{\triangle r}{r}$,	Rel. mag. of SD.
N,	U,	Standard stimulus.
V,	V,	Stim. of comparison.
M,	M,	Arithmetical mean.
MV,	mv,	Mean variation.
$\dfrac{MV}{r}$,	$\dfrac{mv}{r}$,	Rel. del. of S.
$\dfrac{MV}{\triangle r}$, $\dfrac{MV}{N}$,	$\dfrac{mv}{\triangle r}$, $\dfrac{mv}{U}$,	Rel. del. of SD.
$\overset{v}{r}$ or $\triangle r$,	$\overset{v}{r}$ or $\overset{v}{D}$,	Subliminal r or D.
$\hat{r}$ or $\triangle \hat{r}$,	$\hat{r}$ or $\hat{D}$,	Supraliminal r or D.
$|||$,	$|||$,	Subjective $=$.
$\geqq$,	$\geqq$,	Subjective $>$.
$\leqq$,	$\leqq$,	Subjective $<$.
E,	S,	Sensitivity.
UE,	SD,	Sensible discrimination.

The method of just noticeable stimuli requires the further signs r_{λ_o} and r_{λ_u}. That of minimal changes requires $\triangle r_o$, $\triangle r_u$, r_o, r_u and their variants, together with R and $\triangle$. The relation limen[2] I propose to express by Λr; so that

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[1] I use λ, and not l, to avoid confusion of the l with the numeral 1 in printing and black-board writing.

[2] I choose Λ as the initial letter of the Greek λόγος, ratio.

$$\frac{r_{\text{o}}}{r} = \varDelta r_{\text{o}}, \quad \frac{r}{r_{\text{u}}} = \varDelta r_{\text{u}}, \text{ and } \sqrt{\varDelta r_{\text{o}} \cdot \varDelta r_{\text{u}}} = \sqrt{\frac{r_{\text{o}}}{r_{\text{u}}}} = \varDelta r.$$

The methods of equivalents and of mean gradations introduce no new expressions.

In the method of right and wrong cases we set out from a D that is very little $\lessgtr \triangle r$, and obtain the equation:

$$\frac{r}{n} + \frac{w}{n} + \frac{e}{n} + \frac{d}{n} = 1;$$

where r is right, w wrong, e equal and d doubtful, and n denotes the number of observations. The use of r in this method has bred no confusion in German statements of it, and need not do so in English.

For the error stimuli in the method of average error I use the symbol r_{ε}. We have, therefore:

$$\frac{r_{\varepsilon_1} + r_{\varepsilon_2} + r_{\varepsilon_3} \quad \cdot \quad \cdot \quad \cdot \quad r_{\varepsilon_{\text{n}}}}{n} = R_{\varepsilon}.$$

The German $\triangle m$ then becomes mv_{ε}, and $\triangle m_1$ becomes me_{ε}. The English terms suggest what the values really are: the mean variation and average error of the error stimuli obtained.[1]

I subjoin Kuelpe's final Table (p. 78) and the corresponding list of English signs. The h which appears in both tables is Gauss' measure of precision.

GERMAN.

Stimulus and Difference			Determination	Comparison
E.		Magnitude	German S.	$r \parallel r_1$ $(E{:}E_1 = r_1{:}r)$
		Delicacy	MV; MF	MV; MF
U. E.	Mag.	Absolute	S	$(UE : \dfrac{\triangle r \parallel \triangle r_1}{UE_1 = \triangle r_1 : \triangle r})$
		Relative	$\dfrac{S}{r}$	$\dfrac{\triangle r}{r}, \dfrac{\triangle r_1}{r_1};\ r{:}r_1 = r_1{:}r_2$
	Del.	Absolute	MV; MF; h; $\triangle m$	MV; MF
		Relative	$\dfrac{MV}{r}$; $\dfrac{MF}{r}$, $h.r$; $\dfrac{\triangle m}{r}$	$\dfrac{MV}{r}$; $\dfrac{MF}{r}$; $\dfrac{MV}{\triangle r}$, $\dfrac{MF}{\triangle r}$

[1] For the r_{ε} sign series I am indebted to Mr. I. M. Bentley, who has kindly revised this paper in MS. The Greek ε is taken to avoid the symbol me_e.

ENGLISH.

Stimulus and Difference			Determination	Comparison
S.		Magnitude	r_λ	$r \parallel r_1$ $(S : S_1 = r_1 : r)$
		Delicacy	$mv;\ me$	$mv;\ me$
SD.	Mag.	Absolute	$\triangle r$	$D \parallel D_1$ $(SD : SD_1 = D_1 : D)$
		Relative	$\dfrac{\triangle r}{r}$	$\dfrac{D}{r},\ \dfrac{D_1}{r_1};\ r : r_1 = r_1 : r_2$
	Del.	Absolute	$mv;\ me;\ h;\ mv_\varepsilon,\ me_\varepsilon$	$mv;\ me$
		Relative	$\dfrac{mv}{r};\ \dfrac{me}{r};\ h.r;\ \dfrac{mv_\varepsilon}{r},\ \dfrac{me_\varepsilon}{r}$	$\dfrac{mv}{r};\ \dfrac{me}{r};\ \dfrac{mv}{D};\ \dfrac{me}{D}$

I shall greatly value any criticism of these proposals, as well as of the plan of issuing method leaflets. I shall also be glad to know whether laboratory psychologists would welcome the publication of blank sheets prepared for the recording of method work in the drill course.[1] The following, e. g., shows such a sheet prepared for the method of just noticeable stimuli :

" Experimenter,   Unit,
   Subject,   Date,
   Instrument,   Hour,
                     Series   .

Initial value of $\overset{v}{r}$,   Size of steps,   Value of r_{λ_0} ,
Initial value of $\hat{r}$,   Size of steps,   Value of r_{λ_u} ,

                     r_λ =   .

               Mag. of S for   stimuli =   .

Remarks :
References."

[1] Since the sending of this paper to the printer, I have received Vol. IV of the Yale Studies, in which Dr. Scripture describes certain of the experiments given in his drill-course. The note-book made up by the Yale students has some points of resemblance to that which would be made up from my printed blanks; but the two differ very considerably both in method and in object. I therefore leave the above paragraphs as they were written. It may be that the near future will see the publication of a number of laboratory note-books, differing as current text-books differ. The result would be, on the whole, a distinct gain to experimental psychology; the place of the science in the undergraduate course could be better defined, and the undergraduate training of the candidate for graduate scholarships better estimated than is now possible.

Taking Kuelpe's illustration (p. 55), we can fill out the blanks thus:

Experimenter, A. B. C.   Unit, 1 mgr.
Subject, X. Y. Z.   Date, Jan. 27, 1898.
Instrument, Minimal Pressure Weights.   Hour, 3 P. M.

$$\text{Series } \overset{v}{r} \text{ to } r_{\lambda}, \hat{r} \text{ to } r_{\lambda}.$$

Initial value of $\overset{v}{r}$, 1 mgr.   Size of steps, 1 mgr.   Value of r_{λ_0}, 4 mgr.

Initial value of $\hat{r}$, 9 mgr.   Size of steps, 1 mgr.   Value of r_{λ_u}, 6 mgr.

$$r_{\lambda} = 5 \text{ mgr.}$$

$$\text{Mag. of } S \text{ for pressure stimuli} = \tfrac{1}{5}.$$

Remarks:
   Weights were applied to tip of forefinger of right hand.
   Same result was reached if the first step in each direetion was made 2 mgr., and the following steps 1 mgr.

   Remember to repeat the whole series, working from $\hat{r}$ to r_{λ}, and then from $\overset{v}{r}$ to r_{λ}, for the elimination of possible constant error.

   Advisable to repeat both series several times; results interfered with by slipping of weights, unevenness of contact, tickling.
   Notice that the mv, measure of delicacy, coincides with unit of instrument. To get a reliable mv, the unit must be made smaller.
   Etc., etc.   Effects of habituation, expectation, predisposition, practice, fatigue.)

References:
   Kuelpe, "Outlines," p. 56.
   Wundt, "Lectures," p. 50.
   Etc., etc.